

M. Axenides^a, L. Perivolaropoulos^b^a *Institute of Nuclear Physics, N.C.R.P.S. Demokritos, 153 10, Athens, Greece*^b *Department of Physics, University of Patras, Greece.*

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The linearity and quietness of the Local ($< 10Mpc$) Hubble Flow (LHF) in view of the very clumpy local universe is a long standing puzzle in standard and in open CDM cosmogony. The question addressed in this paper is whether the antigravity component of the recently discovered dark energy can cool the velocity flow enough to provide a solution to this puzzle. We calculate the growth of matter fluctuations in a flat universe containing a fraction $\Omega_X(t_0)$ of dark energy obeying the time independent equation of state $p_X = w\rho_X$. We find that dark energy can indeed cool the LHF. However the dark energy parameter values required to make the predicted velocity dispersion consistent with the observed value $v_{rms} \simeq 40km/sec$ have been ruled out by other observational tests constraining the dark energy parameters w and Ω_X . Therefore despite the claims of recent qualitative studies dark energy with time independent equation of state can not by itself explain the quietness and linearity of the Local Hubble Flow.

There is mounting evidence [1] during the past three years that even though the geometry of the universe is flat, a large part of its energy density is smooth and exerts a repelling gravitational force (antigravity) leading to an accelerating expansion on cosmological scales. This type of energy density ρ_X (called ‘dark energy’) consists a fraction $\Omega_X(t_0) \equiv \Omega_{0X}$ of the critical energy density and has an equation of state [2] $p_X = w\rho_X$ with $w < -1/3$. The direct evidence for the existence of dark energy comes from distance measurements [4] of type Ia supernovae (SNe, Ia) which indicate that the expansion of the universe is accelerating. Additional evidence comes from the cosmic microwave background (CMB) anisotropy measurements [6] which indicate [7,8] that the fraction of the total energy density is $\Omega_0 = 1.1 \pm 0.07$ while measurements of the matter density fraction indicate [1] that $\Omega_{0M} = 0.35 \pm 0.07$. Thus, there is converging evidence that about 2/3 of the total energy density of the universe is in the form of a smooth component with negative pressure.

The combination of various observational tests leads to a constrain for the parameters Ω_{0X} and w as [9,10,1]

$$0.85 \gtrsim \Omega_{0X} = 1 - \Omega_{0M} \gtrsim 0.55 \quad (1)$$

and

$$-0.5 \gtrsim w \gtrsim -1 \quad (2)$$

at the 99% confidence level. It is therefore important to study additional cosmological effects of the dark energy in an effort to

1. Impose further constraints on Ω_{0X} and w thus constraining microphysical theories that predict values for these parameters

2. Address cosmological puzzles that may require the features of dark energy for their solution.

The linearity and quietness of the Local Hubble Flow (LHF) is one such puzzle [11]. It is based on three remarkable properties of the local ($\lesssim 10Mpc$) velocity field that are not consistent [12] with standard ($\Omega_{0M} = 1$, $\Omega_{0X} = 0$) CDM (SCDM)

1. The linearity of the velocity-distance relation down to small distances ($\sim 1.5Mpc$)
2. The closeness of the global and local rates of expansion
3. The small local velocity dispersion around the Hubble law ($\sigma_v \simeq 40km/sec$ [13])

It has been proposed [14] based on qualitative arguments, that the properties of dark energy can be manifest in the dynamical features of the LHF in such a way as to remedy this discrepancy with SCDM. The arguments of Ref. [14] are based on the definition of a critical distance $r_Q(M, t)$ such that for scales $r > r_Q$ the repulsive force of dark energy dominates over the gravity of a central mass concentration M at time t . Thus, for $r > r_Q$ the total gravitating mass

$$M_{tot} = \frac{4\pi}{3}(1 + 3w)\rho_X r^3 + M \quad (3)$$

becomes negative. According to Ref. [14], at any time t , the dynamics of all scales $r > r_Q$ are dominated by the effects of dark energy. Therefore if a galaxy in the neighborhood of the Local Group, had spent enough time

beyond r_Q could have its peculiar velocity adiabatically cooled enough to be consistent with the observed cold LHF. The arguments of Ref. [14] are qualitative because they are based on an abrupt transition from matter dominated to dark energy dominated scales and no attempt is made to calculate the total velocity growth factor in a two component cosmological setup. Nevertheless, the conclusion of Ref. [14] is that the adiabatic cooling during dark energy domination in the neighborhood of the Local Group is enough to cool the matter induced peculiar velocities to levels consistent with observations [13] for $w = -2/3$, $\Omega_X \simeq 0.7$.

The main goal of this paper is to quantify this proposal by calculating the growth of density and velocity fluctuations in a flat universe ($\Omega_{0X} + \Omega_{0M} = 1$) containing dark energy characterized by Ω_{0X} and constant w . This calculation leads to suppression growth factors for velocity and density perturbations that depend on Ω_{0X} and w . The main question we wish to address using this calculation is the following: Are there values of Ω_{0X} and w consistent with present constraints that can produce sufficient growth suppression leading to the observed linear and quite LHF?

We thus consider a flat, two component cosmological model with matter ($p_M = 0$) and dark energy ($p_X = w\rho_X$). The Friedman equation describing the evolution of the scale factor is

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2[\Omega_{0M}\left(\frac{a}{a_0}\right)^{-3} + (1 - \Omega_{0M})\left(\frac{a}{a_0}\right)^{-3(1+w)}] \quad (4)$$

where we have used the fact that $\rho_X \sim a^{-3(1+w)}$ (a consequence of the continuity equation $T_{\mu;\nu}^\nu = 0$). Setting $a_0 \equiv a(t_0) = 1$ equation (4) takes the form of

$$\dot{a}^2 = H_0^2[\Omega_{0M}a^{-1} + (1 - \Omega_{0M})a^{-1-3w}] \quad (5)$$

The equations describing the evolution of density and velocity fluctuations in this cosmological model can be obtained as usual from the continuity (for density), momentum (for velocity) and Poisson (for gravitational potential) equations [15]. Combining these equations and linearizing with respect to the gravitational potential, density and velocity perturbations we obtain a second order linear differential equation for the density contrast

$$\delta \equiv \frac{\rho_M(x, t) - \bar{\rho}_M(t)}{\bar{\rho}_M(t)} \quad (6)$$

as

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G\bar{\rho}_M(t)\delta = \frac{3H_0^2\Omega_{0M}}{2a^3}\delta \quad (7)$$

For the corresponding peculiar velocity

$$\vec{v}_{pec} \equiv a\frac{d\vec{x}}{dt} = \frac{d\vec{r}}{dt} - \frac{\dot{a}}{a}\vec{r} \quad (8)$$

(where $\vec{x} = \frac{\vec{r}}{a}$ is the comoving coordinate) we obtain

$$\vec{v}_{pec}(\vec{x}, t) = \dot{a}a\frac{\partial \delta}{\partial a}f(\vec{x}) \quad (9)$$

where $f(\vec{x})$ depends on the initial spatial dependence of density fluctuations. Using equation (5) to change variables from t to a , equation (7) becomes

$$\begin{aligned} & a^2[\Omega_{0M} + (1 - \Omega_{0M})a^{-3w}]\frac{\partial^2 \delta}{\partial a^2} + \\ & a\left[\frac{3\Omega_{0M}}{2} + 3(1 - \Omega_{0M})\left(1 - \frac{1+w}{2}\right)a^{-3w}\right]\frac{\partial \delta}{\partial a} \\ & = \frac{3\Omega_{0M}}{2}\delta \end{aligned} \quad (10)$$

The general solution of this equation is a superposition of two modes a growing mode δ_+ and a decaying mode δ_- . Our numerical solution to this equation will be dominated by the growing mode.

Equation (10) may now be solved numerically with appropriate initial conditions to obtain the growth factors for density and velocity fluctuations evolved from

$$a_i = a_{eq} = \frac{4.31 \times 10^{-5}}{h^2\Omega_{0M}} \quad (11)$$

to the present $a_0 = 1$. Assuming an initially small density fluctuation we may set

$$\delta(t_i \simeq 0) = 0 \quad (12)$$

while since matter dominates over dark energy at early times we may set

$$\frac{\partial \delta}{\partial a}(t_i \simeq 0) = 1 \quad (13)$$

since $\delta_+(a) \sim a$ in a flat matter dominated universe.

In the above initial conditions we have ignored spatial dependence and possible constant factors multiplying δ since we are not interested in the absolute values of fluctuations but in the growth factors

$$g_d(\Omega_{0M}, w) \equiv \frac{\delta(t_0, \Omega_{0M}, w)}{\delta(t_{eq}, \Omega_{0M}, w)} \quad (14)$$

$$g_v(\Omega_{0M}, w) \equiv \frac{v_{pec}(t_0, \Omega_{0M}, w)}{v_{pec}(t_{eq}, \Omega_{0M}, w)} \quad (15)$$

We have solved equation (10) with the initial conditions (12) and (13) and we have constructed contour plots for the relative growth factors $\frac{g_d(\Omega_{0M}, w)}{g_d(1, 0)}$ and $\frac{g_v(\Omega_{0M}, w)}{g_v(1, 0)}$ (Figs

1 and 2). Even though each individual growth factor depends on the value of h the *relative* growth factors as defined above are independent of the Hubble constant.

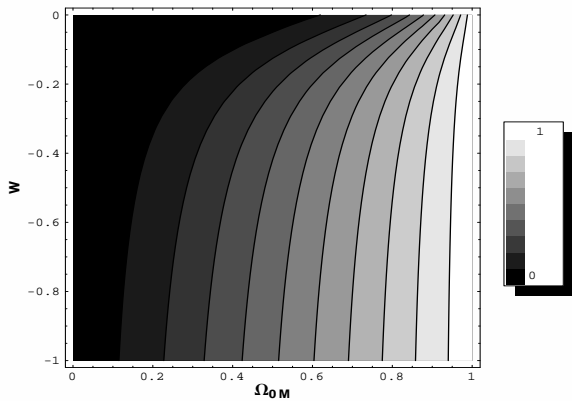


FIG. 1. The relative growth factor $\frac{g_d(\Omega_{0M}, w)}{g_d(1, 0)}$ for density fluctuations for a flat cosmological model as a function of the cosmological parameters Ω_{0M} and w . For small Ω_{0M} and large w the growth gets suppressed as expected due to shorter domination of matter.

These plots indicate that there is a suppression of growth for fluctuations which as discussed in the introduction increases with w and Ω_{0X} . Similar linear growth suppression factors have also been discussed elsewhere [16] and they affect the overall normalization of the power spectrum today. This normalization is expressed by σ_8 , the rms amplitude of matter perturbations on a scale of $8h^{-1}Mpc$. Given a COBE normalized scale invariant cosmological model σ_8 decreases with w , dropping significantly [10] below its presently observed value $\sigma_8 = (0.56 \pm 0.1)\Omega_{0m}^{-0.47}$ for $w > -\frac{1}{2}$ ($\Omega_{0m} \simeq 0.3$). This constraint which becomes slightly stronger for smaller Ω_{0m} has effectively ruled out values of $w > -\frac{1}{2}$ as shown in equation (2).

The deviation from a pure Hubble flow characterized by the observed radial peculiar velocity dispersion is measured to be $v_{rms} \sim 40km/sec$ [13]. An important challenge for cosmological models is the establishment of their consistency with such low velocity dispersion for parameter values that are compatible with other observations. High resolution CDM N body simulations [12] have shown that such low velocity dispersion is incompatible with both standard CDM (SCDM, $\Omega_{0m} = 1$) and open CDM (OCDM, $\Omega_{0m} = 0.3$, $\Omega_{0X} = 0$) which typically predict velocity dispersions

$$\begin{aligned} v_{rms}(SCDM) &\sim 300 - 700km/sec \\ v_{rms}(OCDM) &\sim 150 - 300km/sec \end{aligned} \quad (16)$$

respectively. In fact, according to these simulations, neither of these models can produce a single candidate LG with the observed velocity dispersion in a volume

$10^6 Mpc^3$.

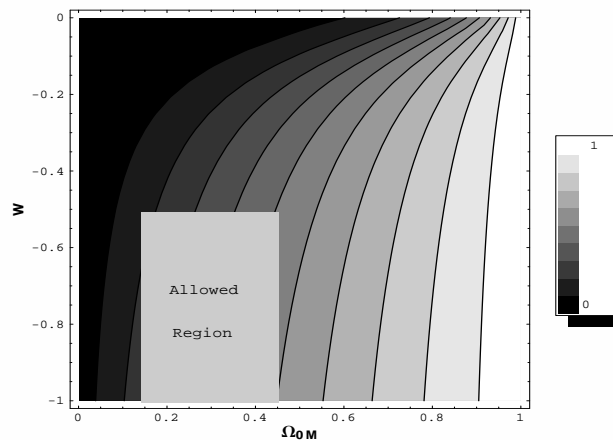


FIG. 2. The relative growth factor $\frac{g_v(\Omega_{0M}, w)}{g_v(1, 0)}$ for velocity fluctuations for a flat cosmological model as a function of the cosmological parameters Ω_{0M} and w .

The above N-body simulations indicate that a suppression factor

$$\bar{g}_v \equiv \frac{g_v(\Omega_{0M}, w)}{g_v(1, 0)} \in [0.06, 0.13] \quad (17)$$

over the SCDM predicted velocity dispersion is required to make the model predictions consistent with observations. Inspection of the suppression factor contour plot of Fig.2 indicates that the presence of dark energy can not by itself provide a solution to this problem. The required values of the suppression factor are achieved within the first and part of the second darkest stripes of Fig. 2. However, no part of this parameter space is consistent with constraints obtained from other observational tests which seem to indicate [9] that $\Omega_{0M} \in [0.15, 0.45]$ and $w \in [-1, -0.5]$ at 99% level.

We conclude that the presence of dark energy with time independent equation of state can not by itself resolve the puzzle of the LHF linearity and quietness by suppressing the growth of velocity fluctuations. The required velocity dispersion suppression is achieved for parameter values that are incompatible with other observational tests and requires values of Ω_{0M} that are too low (e.g. $w \lesssim -0.5$ leads to $\Omega_{0M} \simeq 0.04$) out of the currently allowed interval. Alternatively for a fixed value of $\Omega_{0M} \simeq 0.3$, the resolution of the LHF puzzle with dark energy requires too large w ($w \gtrsim -0.3$). These values are inconsistent with both the recent CMB data from MAXIMA and BOOMERanG l experiments which seem to favor [1,10] a value of $w \simeq -0.6$ for $\Omega_{0M} \simeq 0.3$ and the presently observed values of σ_8 .

An interesting extension of our work could be the introduction of a time dependence in the dark energy equation of state along the lines of quintessence models *i.e.*

$p = (w_0 + w_1 z)\rho$. This type of equation of state allows for non-trivial effects of dark energy even at early times which could lead to a similar suppression of velocity fluctuation growth at higher values of Ω_{0M} and/or lower values of w thus being consistent with current observational constraints.

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